RADIATIVE TRANSFER FROM A POINT SOURCE THROUGH A SCATTERING LAYER

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By means of an approximate method we analyze the relationships which exist between the optical thickness of a layer and the coefficients of diffuse radiative transmission and reflection, as well as an analysis of the experimental conditions.

The solution of the two-dimensional problem of the propagation of radiation from a point source through a scattering layer is of considerable significance in the case of radiative heat exchange, the spectroscopy of scattering media, atmospheric optics, neutron transport, and other branches of physics. To derive this solution we generally make use of the methods of direct numerical integration of the transport equation, the method of spherical harmonics, the Monte Carlo method, etc. [1-4]. However, direct utilization of the derived data, as cited in the literature, for purposes of engineering calculations is difficult in the majority of cases. In the present paper we reduce the stated problem to the familiar problem of radiative transfer in a layer on which a plane-parallel bundle of external radiation is incident [5]. It is demonstrated that the utilization of the proposed method leads to errors that are quite satisfactory from the standpoint of estimating the radiation of the spot source, transmitted or reflected by the scattering layer.

Let a layer of a scattering medium with a thickness $(z_1 - z_0)$ be characterized by absorption indices κ and scattering indices σ [$\lambda = \sigma/(\kappa + \sigma)$ represents the probability of quantum survival]. The positive direction of the z axis is set in a direction away from the source, perpendicular to the surface of the layer (Fig. 1a). We will assume the scattering to be isotropic. The anisotropy of the scattering may be taken into consideration by the method developed in [6, 7]. At a distance z_0 from the layer we find a radiation point source of power S_0. The problem is symmetrical about the OZ axis, and we will therefore limit ourselves to an examination of this problem in the ZOY plane (Fig. 1b).

Let us turn to the problem of the transmission and reflection of radiation by a plane layer of finite optical thickness $\tau_0 = (\kappa + \sigma)z_0$ irradiated by a bundle of a certain power, at some angle. As we know [5], in this case the coefficients of diffuse transmission and reflection for the radiation are determined by the following expressions:

$$\sigma(\tau_0, \mu, \mu_0) = \frac{I(\tau_0, \mu, \mu_0)}{\mu_0 S_0} = \frac{\lambda}{4}\frac{\varphi(\mu_0)\psi(\mu) - \psi(\mu_0)\varphi(\mu)}{\mu - \mu_0},$$
$$\rho(\tau_0, \mu, \mu_0) = \frac{I(0, \mu, \mu_0)}{\mu_0 S_0} = \frac{\lambda}{4}\frac{\varphi(\mu_0)\varphi(\mu) - \psi(\mu_0)\psi(\mu)}{\mu + \mu_0}, \tag{1}$$

where $\varphi(\mu)$ and $\psi(\mu)$ are the Ambartsumyan functions; $\theta_0 = \arccos\mu_0$ and $\theta = \arccos\mu$ determine the directions of incidence of external radiation and the observation of the transmitted or reflected radiation. The Ambartsumyan functions are calculated from a system of integral equations and at the present time these have been tabulated in considerable detail for a number of physical situations (accounting for anisotropy, the phenomena of redistribution of radiation by frequency, etc.) (see, for example [5, 8, 9]).

Let us choose some angle γ (on the order of 5-20°) and let us examine the diagram shown in Fig. 1b. The rays S_0A', S_0A, S_0B, ..., combining to form the angle γ, divide a plane layer

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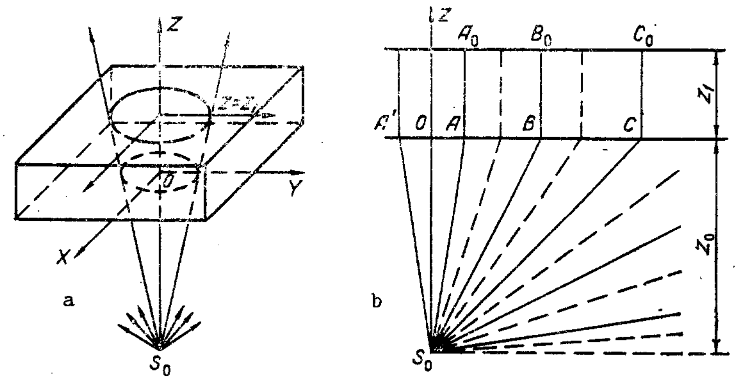

Fig. 1. Calculating the reflection and transmission of the radiation emitted from a point source.

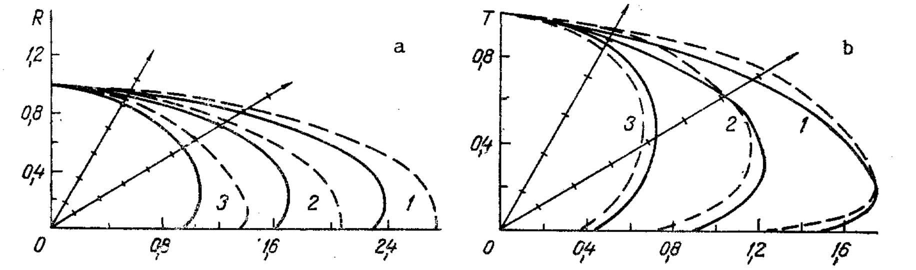

Fig. 2. Angular distribution of reflected (a) and transmitted (b) radiation when $\gamma = 10°$ for $n = 0$ (solid lines) and $n = 6$ (dashed lines): 1) $\tau_0 = 0.6$; 2) 1.0; 3) 3.0.

TABLE 1. Values of the Ambartsymyan Functions $\varphi(\tau_0, \mu)$ and $\psi(\tau_0, \mu)$ for the Conservative Case [8]

μ	τ_0											
	0,2		0,66		1,0		2,0		3,0		∞	
	φ	ψ	φ	ψ	φ	ψ	φ	ψ	φ	ψ	φ	ψ
0,1	1,147	0,241	1,182	0,075	1,194	0,056	1,211	0,037	1,219	0,028	1,247	0
0,2	1,198	0,534	1,293	0,213	1,327	0,137	1,365	0,086	1,385	0,066	1,450	0
0,3	1,222	0,711	1,368	0,379	1,429	0,250	1,498	0,149	1,531	0,112	1,643	0
0,4	1,236	0,823	1,421	0,530	1,510	0,376	1,613	0,224	1,663	0,167	1,829	0
0,5	1,245	0,899	1,460	0,657	1,574	0,500	1,715	0,311	1,785	0,231	2,013	0
0,6	1,251	0,953	1,489	0,762	1,626	0,615	1,805	0,404	1,895	0,303	2,194	0
0,7	1,256	0,995	1,512	0,850	1,668	0,719	1,884	0,501	1,997	0,381	2,374	0
0,8	1,259	1,027	1,531	0,923	1,703	0,811	1,954	0,597	2,090	0,464	2,553	0
0,9	1,262	1,053	1,546	0,986	1,732	0,893	2,016	0,691	2,175	0,549	2,731	0
1,0	1,265	1,074	1,558	1,039	1,757	0,966	2,071	0,780	2,254	0,636	2,908	0

in the direction of the OY axis into a number of finite layers AA', AB, BC, ... To solve the stated problem we will assume that these layers are characterized by certain values for the transmission coefficients $\sigma_n = \sigma(\tau_0, \mu, \mu_n)$ and for the reflection coefficients $\rho_n = \rho(\tau_0, \mu, \mu_n)$, equal to their infinite analogs. As a consequence of the insignificant difference in the radiation streams at the separating surfaces AA_0, BB_0, CC_0, ..., given a sufficiently large n, such a representation is entirely satisfactory. There is no doubt that the error depends on the magnitude of γ. With small γ values the interaction of the isolated adjacent layers is quite strong, but in this case the intensities of radiation incident on the corresponding layers will differ insignificantly from each other, and this weakens the indicated interaction. As γ is increased, the interaction between the layers (with the exception of the boundary region between the layers) is weakened due to the larger value $(\kappa + \sigma) \cdot \Delta y_n = (\kappa + \sigma)(y_{n+1} - y_n)$, but in this case the error is increased because of the disruption of the plane-parallel condition under which the external radiation is incident. Hence follows the need to study the unique features of the relationships σ_n and ρ_n ($n = 0, 1, 2, \ldots, N$; $N = \pi/2\gamma$) to the quantity γ. For conservative systems ($\lambda = 1$) the criterion of validity for the derived solution can be found in the equality

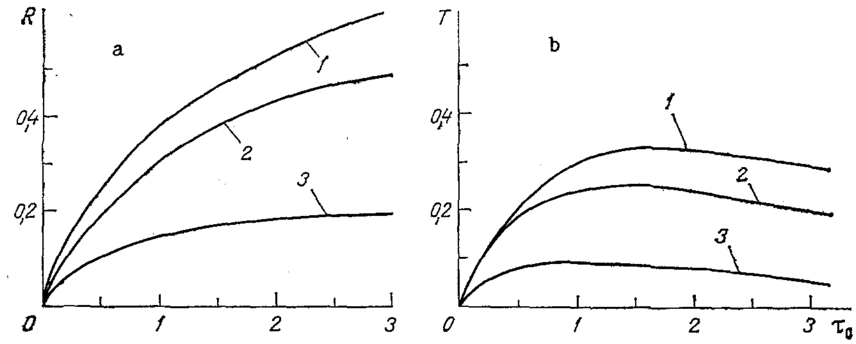

Fig. 3. Reflected (a) and transmitted (b) radiation as functions of the optical thickness of the layer for various zones: 1) n = 0; 2) 3; 3) 6.

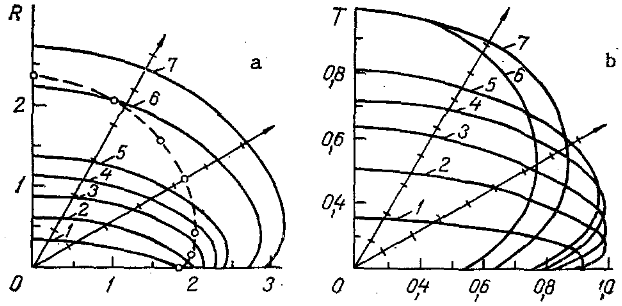

Fig. 4. Reflectance (a) and transmittance (b) of the scattering layer, with various optical thicknesses: 1) τ_0 = 0.2; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.0; 6) 2.0; 7) 3.0 ($\tau_0 \to \infty$ is represented by the dashed curve, and the data have been reduced by a factor of two).

$$\int_0^1 \left\{\sigma_0 + \rho_0 + 2(\sigma_N + \rho_N) + 2\sum_{n=1}^{N-1} (\sigma_n + \rho_n)\right\} d\mu = A(\tau_0), \tag{2}$$

where σ_n and ρ_n (n = 0, 1, 2, ..., N − 1) is calculated in accordance with (1), while for σ_N and ρ_N we assume the values of σ and ρ when $\mu_n = \cos\gamma(N - 1/4) = \sin\gamma/4$. In the case of a semiinfinite layer ($\tau_0 \to \infty$) $\sigma_n \to 0$ (n = 0, 1, 2, ..., N), $A(\tau_0) \to 1$, which may serve as a criterion of validity for the selected model. Another criterion (but now more approximate) is

$$\int_0^1 d\mu \int_0^1 \sigma(\tau_0, \mu, \mu')\, d\mu' + \int_0^1 d\mu \int_0^1 \rho(\tau_0, \mu, \mu')\, d\mu' = A_0(\tau_0) \geqslant A(\tau_0). \tag{3}$$

Inequality (3) follows from the fact that in relationship (2) the quantities σ_n and ρ_n decline markedly as n increases. The accuracy of the proposed method will subsequently be defined more correctly in the case of various physical conditions when compared to the numerical calculations through utilization of the Monte Carlo method.

For each n-th bounded layer ($y_n \le y \le y_{n+1}$, AA' = $2y_0$) it is not difficult to determine the original parameters for the calculation of the transmission coefficients σ_n and the reflection coefficients ρ_n (n = 0, 1, 2, ..., N):

$$\mu_n = \cos\theta_n,\ \theta_n = n\gamma,\ S_n = \frac{S_0}{z_0^2} T_{1n} \frac{1}{2N} \mu_n^2, \tag{4}$$

where T_{1n} is the transmission function for the radiation along the n-th ray from the source S_0 to the n-th layer. In the general case

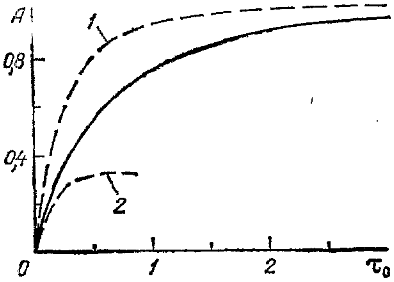

Fig. 5. The $\sigma + \rho = A$ criteria as a function of the optical thickness of the layer: 1) relationship (3); 2) single scattering.

TABLE 2. Relative Values of $A(\tau_0)$ as a Function of γ [the values of $A(\tau_0)$ for $\gamma = 5°$ are taken to be 1]

γ	τ_0				
	0,4	1,0	2,0	3,0	$\tau_0 \to \infty$
5° (N=18)	1,0	1,0	1,0	1,0	1,0
10° (N=9)	0,998	0,998	0,997	0,997	0,991
15° (N=6)	0,995	0,996	0,995	0,994	0,993
18° (N=5)	0,995	0,995	0,993	0,992	0,991

$$T_{1n} = T\left(\frac{\alpha_1 z_0}{\mu_n}\right) \tag{5}$$

(α_1 is the attenuation factor in the medium for the case in which $z < 0$). For a uniform and gray medium with $z < 0$

$$T_{1n} = \exp\left(-\frac{\alpha_1 z_0}{\mu_n}\right). \tag{5a}$$

Then, in accordance with (1) we find that for the region $y_n \le y \le y_{n+1}$ the intensities of the transmitted and reflected radiation at a distance z_2 from the layer in the direction $\theta = \arccos\mu$ are equal to:

$$I^{(n)}_{\text{trans}} = I_n(\tau_0, \mu, \mu_n) = \frac{\lambda}{4}\frac{S_0}{z_2^2} T_2 T_{1n} \frac{1}{2N} \mu_n^3 \frac{\varphi(\mu_n)\psi(\mu) - \psi(\mu_n)\varphi(\mu)}{\mu - \mu_n}, \tag{6}$$

$$I^{(n)}_{\text{rel}} = I_n(0, \mu, \mu_n) = \frac{\lambda}{4}\frac{S_0}{z_2^2} T_3 T_{1n} \frac{1}{2N} \mu_n^3 \frac{\varphi(\mu_n)\varphi(\mu) - \psi(\mu_n)\psi(\mu)}{\mu + \mu_n}, \tag{7}$$

where $T_2 = T(\alpha_2 z_2/\mu)$; $T_3 = T(\alpha_1 z_2/\mu)$; α_2 is the attenuation factor in the medium $z > z_0$. We note that relationships (6) and (7) differ from each other by the factor μ_n^2.

Specific calculations of relationships (6) and (7) have been carried out for the conservative case ($\lambda = 1$), $T_{1n} = T_2 = T_3 = 1$ (the media in the case of $z > 0$ and $z < 0$ are transparent) and γ = 30, 18, 15, and 5°. The values of the Ambartsumyan functions for the conservative case are shown in Table 1, in accordance with the data from [8].

The angular distribution of the radiation emanating out of two zones of the layer ($n = 0$ and $n = 6$), for the case in which $\gamma = 10°$, is shown in Fig. 2. With an increase in τ_0 the anisotropy of the angular distribution diminishes and in the direction perpendicular to the layer ($\theta = 0°$), the radiation may exceed that in the direction "of the edge" ($\theta = 90°$), especially for the transmitted radiation. In terms of absolute magnitude, the intensities of the reflected and transmitted radiation depend variously on the optical thickness of the layer (Fig. 3). For the far zones the reflected radiation (Fig. 3, $n = 6$, shown as curve 3) rather quickly tends to the asymptotic value. For the transmitted radiation, on attaining some maximum, the intensity diminishes, with the intensity of the reflected radiation reaching its maximum value at various values of τ_0 for the various zones. The angular distribution of the zone-summed radiations is shown in Fig. 4. With a change in τ_0 from 0.2 to 3 the anisotropy of the angular distribution

$$r = I|_{\theta=90°}/I|_{\theta=0°}$$

changes from 6.0 to 1.5 for the reflected radiation and from 3.0 to 0.4 for the transmitted radiation.

If the function $A(\tau_0)$ is presented in accordance with relationship (2), then it occupies some intermediate position between relationship (3) and that quantity determined from a single scattering of the radiation (Fig. 5). With an error not exceeding 10-15%, curve 1 in Fig. 5 can be obtained by means of approximate relationships for the Ambartsumyan functions, such as those derived for the conservative scattering medium described by Samson in [10]:

$$\varphi_{ap}(\mu) = \frac{1}{1+\tau_0}\left[(2\mu+1)(1+\tau_0-\mu)+\mu(2\mu-1)\exp\left(-\frac{\tau_0}{\mu}\right)\right],$$

$$\psi_{ap}(\mu) = \frac{1}{1+\tau_0}\left[\mu(2\mu+1)-(2\mu-1)(1+\tau_0+\mu)\exp\left(-\frac{\tau_0}{\mu}\right)\right].$$

It is interesting to note that the derived results depend only slightly on the magnitude of γ selected. This is borne out in Table 2, where we find the calculation data for $A(\tau_0)$ for various values of γ. For $\gamma = 30°$, in calculating the reflected radiation, we find that the error is as high as 4-5%. Subsequently, to establish the accuracy of the proposed method in various physical situations, we conducted a comparison with the calculation data obtained by means of the Monte Carlo method.

NOTATION

$I_n(\tau_0, \mu, \mu_n)$ [or $I_n(0, \mu, \mu_n)$], intensity of layer-transmitted (or layer-reflected) radiation from the n-th zone in the direction $\theta = \arccos\mu$; S_0, the power of the point source; T and R, transmittance and reflectance functions; $\varphi(\mu)$ and $\psi(\mu)$, Ambartsumyan functions; z_0, distance from the radiation source to the layer; z_1, thickness of the layer; $\tau_0 = (\kappa + \sigma)z_1$, optical thickness of the layer; κ and σ, coefficients of medium absorption and scattering; $\lambda = \sigma/(\kappa + \sigma)$, probability of quantum survival.

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